Sensors and state estimation

ST5 Autonomous robotics

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Introduction

Perception

- interpretation of sensor values
- inference on the environment
- inference on the state of the robot
- building of an internal representation

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Aim of this session

- presentation of various kinds of sensors
- introduction to state estimation

01

Sensors

Definitions

Sensor

- physical device
- measuring some physical phenomenon
- in a particular region of space

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Characteristics

- view angle, range, frequency
- accuracy (bias), precision (variability)
- drift, saturation
- weight, active/passive, power draw...

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Two kinds

- proprioceptive: information on the robot itself
- exteroceptive: information on the environment

Distance sensors

Sonar

- time of flight of ultrasound pulse (40–68 kHz)
- range of a few meters, angle of a few dozens of degrees
- 10–25 Hz ($\sim$18 ms for 3 m round trip)
- not great on cloth

Devantech SRF02

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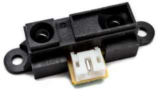
Devantech SRF02

Infrared

- intensity or angle of an infrared pulse (800–900 nm)
- range around a meter, angle of a few degrees
- $\sim$20 Hz
- not great on mate black



Sharp GP2Y0A21YK0F

Distance sensors

Unidirectional laser

- time of flight of a laser pulse
- dozens of meters, very focused
- $\sim$20 Hz
- not great on reflective surfaces

Lightwave SF02

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Lightwave SF02

Laser scanner

- time of flight, rotative sensor (mirror)
- 180–270–360° scanning angle with 360–1080 points, 4–80 m
- 20–50 Hz
- expensive, heavy

Hokuyo UTM30-LX

Distance sensors

Rotating laser

- time of flight of a laser pulse
- ∼100 m, 360° horizontal, ∼30° vertical with several channels (16, 32, 64)
- ∼1 Mpts/s, ∼10 rev/s
- big, expensive, heavy

Velodyne HDL-64E

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Velodyne HDL-64E

Time of flight cameras

- time of flight of IR pulse with matrix of sensors
- several meters
- 30–60 Hz
- not great outside

Mesa Imaging SR4000

Cameras

Color camera

- quantity of light on color receptors
- angle of view ∼10–100°, unconstrained range
- small, light, low power, cheap
- difficult to calibrate

Random camera
(VC0706 UART VGA)

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Random camera
(VC0706 UART VGA)

Omnidirectional camera

- several cameras
- lens
- mirror
- difficult to calibrate

Immersive Media Dodeca 2360

Kodak Pixpro SP360

0-360 Panoramic Optic

Depth cameras

Stereo camera

- disparity between two images
- decreasing precision with distance
- not great with uniform textures

PointGrey Bumblebee2

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Stereo camera

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PointGrey Bumblebee2

RGB-D camera

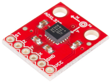
- color camera + depth
- stereo with structured light projector or time of flight
- calibration between RGB and D

Asus Xtion Pro

Inertial Measurement Unit (IMU)

Accelerometer

- measure *proper acceleration* along a given axis
- hundreds of Hz
- drift
- small, cheap, low power (MEMS)



Sparkfun ADXL335

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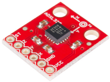

Sparkfun ADXL335

Gyroscope

- angular velocity
- hundreds of Hz
- drift
- can be small, cheap and low power (MEMS)

Sparkfun ITG-3200

Other sensors

Other sensors

- wheel encoders (can be embedded with the motors)
- force
- switch
- temperature
- humidity
- pressure...

02

State estimation

State estimation

State estimation

- compute an estimate of the state of the robot
- from sensors values
- one of the perception problems
- needs sensor models
- needs a robot model

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Approaches

- signal processing
- Bayesian filtering
- Kalman filtering

Bayesian filter

Model

$$p(\boldsymbol{x}_{0:T}, \boldsymbol{z}_{1:T}, \boldsymbol{u}_{1:T}) = p(\boldsymbol{x}_0) \prod_{k=1}^{T} p(\boldsymbol{u}_k) p(\boldsymbol{x}_k \mid \boldsymbol{x}_{k-1}, \boldsymbol{u}_k) p(\boldsymbol{z}_k \mid \boldsymbol{x}_k)$$

Bayesian filter

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Inference

$$p(\boldsymbol{x}_k \mid \boldsymbol{z}_{1:k}, \boldsymbol{u}_{1:k})$$

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$$p(\boldsymbol{x}_k \mid \boldsymbol{z}_{1:k}, \boldsymbol{u}_{1:k})$$

$$p(\boldsymbol{x}_{k+1} \mid \boldsymbol{z}_{1:k}, \boldsymbol{u}_{1:k+1}) = \sum_{\boldsymbol{x}_k} p(\boldsymbol{x}_{k+1} \mid \boldsymbol{x}_k, \boldsymbol{u}_{k+1}) p(\boldsymbol{x}_k \mid \boldsymbol{z}_{1:k}, \boldsymbol{u}_{1:k})$$

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$$p(\boldsymbol{x}_{k+1} \mid \boldsymbol{z}_{1:k+1}, \boldsymbol{u}_{1:k+1}) \propto p(\boldsymbol{z}_{k+1} \mid \boldsymbol{x}_{k+1}) p(\boldsymbol{x}_{k+1} \mid \boldsymbol{z}_{1:k}, \boldsymbol{u}_{1:k+1})$$

Kalman filter

Overview on Kalman filtering

- Gaussian probability distributions
- linear transition and observation models

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Variables

- state vector: $\boldsymbol{x}_k$
- observation vector: $\boldsymbol{z}_k$
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Models

Transition

$$\boldsymbol{x}_k = \boldsymbol{F}_k \boldsymbol{x}_{k-1} + \boldsymbol{B}_k \boldsymbol{u}_k + \boldsymbol{w}_k$$

Observation

$$\boldsymbol{z}_k = \boldsymbol{H}_k \boldsymbol{x}_k + \boldsymbol{v}_k$$

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$$p(\boldsymbol{x}_k \mid \boldsymbol{x}_{k-1}, \boldsymbol{u}_k) = \mathcal{N}(\boldsymbol{F}_k \boldsymbol{x}_{k-1} + \boldsymbol{B}_k \boldsymbol{u}_k, \boldsymbol{Q}_k)$$

Observation

$$\boldsymbol{z}_k = \boldsymbol{H}_k \boldsymbol{x}_k + \boldsymbol{v}_k$$

$$p(\boldsymbol{z}_k \mid \boldsymbol{x}_k) = \mathcal{N}(\boldsymbol{H}_k \boldsymbol{x}_k, \boldsymbol{R}_k)$$

Inference in a Kalman filter

Principle

- closed form for exact inference
- distributions represented by mean and covariance: $\hat{\boldsymbol{x}}_{k|k}$, $\boldsymbol{P}_{k|k}$

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Prediction

$$\hat{\boldsymbol{x}}_{k|k-1} = \boldsymbol{F}_k \hat{\boldsymbol{x}}_{k-1|k-1} + \boldsymbol{B}_k \boldsymbol{u}_k$$
$$\boldsymbol{P}_{k|k-1} = \boldsymbol{F}_k \boldsymbol{P}_{k-1|k-1} \boldsymbol{F}_k^\mathsf{T} + \boldsymbol{Q}_k$$

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$$\boldsymbol{P}_{k|k-1} = \boldsymbol{F}_k \boldsymbol{P}_{k-1|k-1} \boldsymbol{F}_k^\intercal + \boldsymbol{Q}_k$$

Update / Correction

$$\tilde{\boldsymbol{y}}_k = \boldsymbol{z}_k - \boldsymbol{H}_k \hat{\boldsymbol{x}}_{k|k-1}$$

$$\boldsymbol{S}_k = \boldsymbol{H}_k \boldsymbol{P}_{k|k-1} \boldsymbol{H}_k^\intercal + \boldsymbol{R}_k$$

$$\boldsymbol{K}_k = \boldsymbol{P}_{k|k-1} \boldsymbol{H}_k^\intercal \boldsymbol{S}_k^{-1}$$

$$\hat{\boldsymbol{x}}_{k|k} = \hat{\boldsymbol{x}}_{k|k-1} + \boldsymbol{K}_k \tilde{\boldsymbol{y}}_k$$

$$\boldsymbol{P}_{k|k} = (I - \boldsymbol{K}_k \boldsymbol{H}_k) \boldsymbol{P}_{k|k-1}$$

State estimation

Example

- altitude estimation of a blimp
- with a sonar
- no command

Variables

- x: altitude
- z: distance to ground measured by the sonar

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- $\boldsymbol{x}$: altitude
- $\boldsymbol{z}$: distance to ground measured by the sonar

Parameters

- $\forall k, \boldsymbol{F}_k = \boldsymbol{F} = 1$
- $\forall k, \boldsymbol{Q}_k = \boldsymbol{Q} = 0.01^2 \text{ m}^2$
- $\forall k, \boldsymbol{H}_k = \boldsymbol{H} = 1$
- $\forall k, \boldsymbol{R}_k = \boldsymbol{R} = 0.05^2 \text{ m}^2$

Example

Initialization

$$\hat{\boldsymbol{x}}_{0|0} = 1$$

$$\boldsymbol{P}_{0|0} = 0.2^2\ \mathsf{m}^2$$

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$$\hat{\boldsymbol{x}}_{k|k-1} = \boldsymbol{F}_k \hat{\boldsymbol{x}}_{k-1|k-1} + \boldsymbol{B}_k \boldsymbol{u}_k$$
$$\boldsymbol{P}_{k|k-1} = \boldsymbol{F}_k \boldsymbol{P}_{k-1|k-1} \boldsymbol{F}_k^T + \boldsymbol{Q}_k$$

Example

Initialization

$$\hat{\boldsymbol{x}}_{0|0} = 1$$
$$\boldsymbol{P}_{0|0} = 0.2^2 \text{ m}^2$$

Prediction

$$\hat{\boldsymbol{x}}_{1|0} = 1$$
$$\boldsymbol{P}_{1|0} = 0.0401$$

$$\hat{\boldsymbol{x}}_{k|k-1} = \boldsymbol{F}_k \hat{\boldsymbol{x}}_{k-1|k-1} + \boldsymbol{B}_k \boldsymbol{u}_k$$
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Correction with $\boldsymbol{z}_1 = 0.8$

$$\tilde{\boldsymbol{y}}_k = \boldsymbol{z}_k - \boldsymbol{H}_k \hat{\boldsymbol{x}}_{k|k-1}$$
$$\boldsymbol{S}_k = \boldsymbol{H}_k \boldsymbol{P}_{k|k-1} \boldsymbol{H}_k^T + \boldsymbol{R}_k$$
$$\boldsymbol{K}_k = \boldsymbol{P}_{k|k-1} \boldsymbol{H}_k^T \boldsymbol{S}_k^{-1}$$
$$\hat{\boldsymbol{x}}_{k|k} = \hat{\boldsymbol{x}}_{k|k-1} + \boldsymbol{K}_k \tilde{\boldsymbol{y}}_k$$
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Example

Prediction

$$\hat{\boldsymbol{x}}_{1|0} = 1 \qquad \hat{\boldsymbol{x}}_{k|k-1} = \boldsymbol{F}_k \hat{\boldsymbol{x}}_{k-1|k-1} + \boldsymbol{B}_k \boldsymbol{u}_k$$

$$\boldsymbol{P}_{1|0} = 0.0401 \qquad \boldsymbol{P}_{k|k-1} = \boldsymbol{F}_k \boldsymbol{P}_{k-1|k-1} \boldsymbol{F}_k^T + \boldsymbol{Q}_k$$

Correction with $\boldsymbol{z}_1 = 0.8$

$$\tilde{\boldsymbol{y}}_1 = -0.2 \qquad \tilde{\boldsymbol{y}}_k = \boldsymbol{z}_k - \boldsymbol{H}_k \hat{\boldsymbol{x}}_{k|k-1}$$

$$\boldsymbol{S}_1 = 0.0426 \qquad \boldsymbol{S}_k = \boldsymbol{H}_k \boldsymbol{P}_{k|k-1} \boldsymbol{H}_k^T + \boldsymbol{R}_k$$

$$\boldsymbol{K}_1 = 0.9413 \qquad \boldsymbol{K}_k = \boldsymbol{P}_{k|k-1} \boldsymbol{H}_k^T \boldsymbol{S}_k^{-1}$$

$$\hat{\boldsymbol{x}}_{1|1} = 0.8117 \qquad \hat{\boldsymbol{x}}_{k|k} = \hat{\boldsymbol{x}}_{k|k-1} + \boldsymbol{K}_k \tilde{\boldsymbol{y}}_k$$

$$\boldsymbol{P}_{1|1} = 0.0024 \qquad \boldsymbol{P}_{k|k} = (I - \boldsymbol{K}_k \boldsymbol{H}_k) \boldsymbol{P}_{k|k-1}$$

Example

Correction with $\boldsymbol{z}_1 = 0.8$

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Prediction

$$\hat{\boldsymbol{x}}_{2|1} = 0.8117 \qquad \hat{\boldsymbol{x}}_{k|k-1} = \boldsymbol{F}_k \hat{\boldsymbol{x}}_{k-1|k-1} + \boldsymbol{B}_k \boldsymbol{u}_k$$

$$\boldsymbol{P}_{2|1} = 0.0025 \qquad \boldsymbol{P}_{k|k-1} = \boldsymbol{F}_k \boldsymbol{P}_{k-1|k-1} \boldsymbol{F}_k^T + \boldsymbol{Q}_k$$

Example

Prediction

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$$\boldsymbol{P}_{2|1} = 0.0025 \qquad \boldsymbol{P}_{k|k-1} = \boldsymbol{F}_k \boldsymbol{P}_{k-1|k-1} \boldsymbol{F}_k^T + \boldsymbol{Q}_k$$

Correction with $\boldsymbol{z}_2 = 0.85$

$$\tilde{\boldsymbol{y}}_2 = 0.0383 \qquad \tilde{\boldsymbol{y}}_k = \boldsymbol{z}_k - \boldsymbol{H}_k \hat{\boldsymbol{x}}_{k|k-1}$$

$$\boldsymbol{S}_2 = 0.0050 \qquad \boldsymbol{S}_k = \boldsymbol{H}_k \boldsymbol{P}_{k|k-1} \boldsymbol{H}_k^T + \boldsymbol{R}_k$$

$$\boldsymbol{K}_2 = 0.4953 \qquad \boldsymbol{K}_k = \boldsymbol{P}_{k|k-1} \boldsymbol{H}_k^T \boldsymbol{S}_k^{-1}$$

$$\hat{\boldsymbol{x}}_{2|2} = 0.8307 \qquad \hat{\boldsymbol{x}}_{k|k} = \hat{\boldsymbol{x}}_{k|k-1} + \boldsymbol{K}_k \tilde{\boldsymbol{y}}_k$$

$$\boldsymbol{P}_{2|2} = 0.0012 \qquad \boldsymbol{P}_{k|k} = (I - \boldsymbol{K}_k \boldsymbol{H}_k) \boldsymbol{P}_{k|k-1}$$

03

Conclusion

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- physical measurement process is important

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Kalman filter

- estimation of mean and covariance
- linear Gaussian models (extensions: EKF, UKF, particle filter...)

Bibliography

Books

- Thrun *et al.*, *Probabilistic Robotics*, MIT Press, 2005.
- Siegwart *et al.*, *Introduction to Autonomous Mobile Robots*, MIT Press, 2011.
- Siciliano *et al.*, *Springer Handbook of Robotics*, 2nd ed., Springer, 2016.

Wikipedia

- `https://en.wikipedia.org/wiki/Kalman_filter`

Thanks for your attention

Questions?