Motion planning

ST5 Autonomous robotics

Francis Colas

2022-09-23

Introduction

Motion

- generated by motors
  - electric (AC/DC), pneumatic or hydraulic
  - stepper motor: fixed positions
  - servomotors: motor+integrated sensor for position control
- controlled to perform a given trajectory

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- while following constraints (collision avoidance, dynamics...)

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- while following constraints (collision avoidance, dynamics...)

Aim of the session

- configuration space
- planning algorithms

01

Configuration space

Trajectory

Path

- sequence of poses
- typically from a start to a goal location

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Trajectory

- mapping from time to configuration
- allows to compute velocities and commands

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- allows to compute velocities and commands

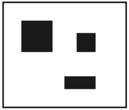
Constraints

- minimum length/distance
- minimum cost (application-dependent)
- security/distance to obstacles
- kinematics or dynamics

Configuration space

Workspace $\mathcal{W}$

- space in which robot evolves
  - in general: $\mathcal{W} = \mathbb{R}^3$
  - sometimes: $\mathcal{W} = \mathbb{R}^2$



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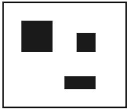
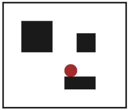


Configuration space $\mathcal{C}$

- set of feasible configurations $\boldsymbol{q} \in \mathcal{C}$
- taking into account physical constraints
- in general $\mathcal{C} \neq \mathcal{W}$



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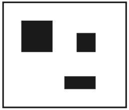

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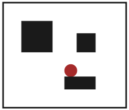
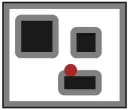


Free space $\mathcal{E}$

- configurations not in collision
- $\mathcal{E} = \mathcal{C} \setminus \mathcal{O}$
- adapted for planning



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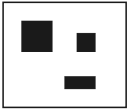

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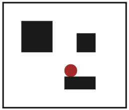

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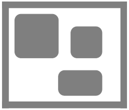
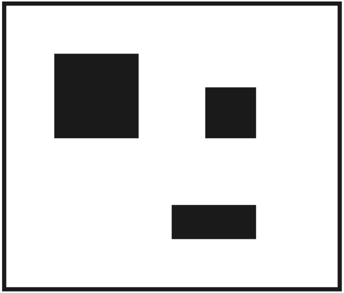


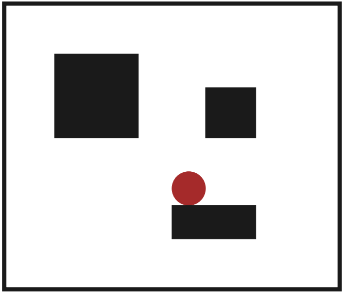
Holonomic circular mobile robot

Workspace



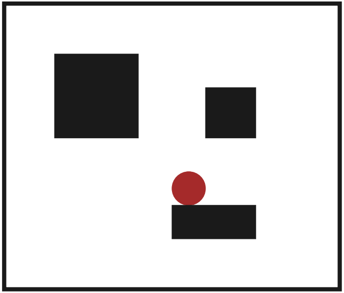
Holonomic circular mobile robot

Workspace and robot

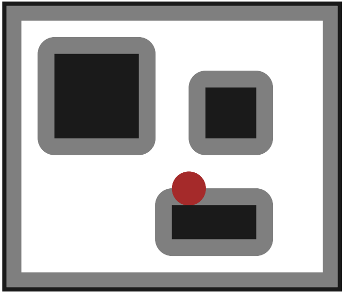


Holonomic circular mobile robot

Workspace and robot

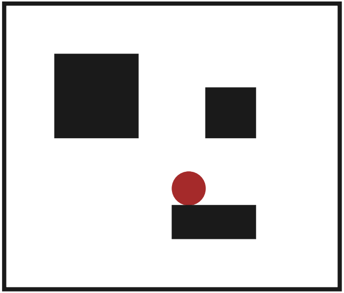


Obstacle inflation (Minkowski sum)



Holonomic circular mobile robot

Workspace and robot



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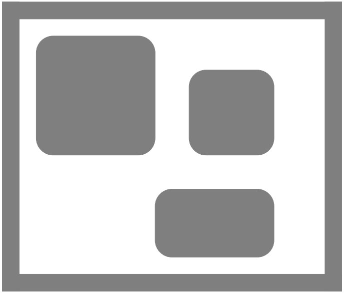
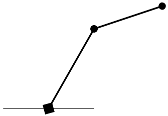


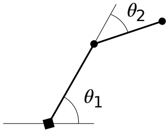
Robotic arm

Robotic arm

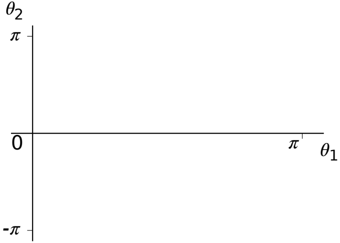


Robotic arm

2 degrees-of-freedom arm



Degrees of freedom



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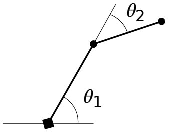
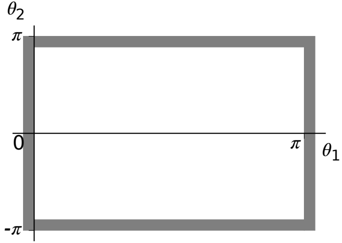
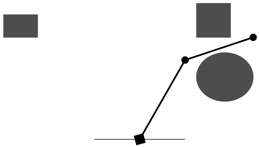


Configuration space

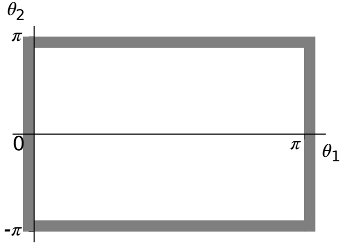


Robotic arm

Workspace



Configuration space



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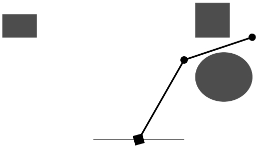
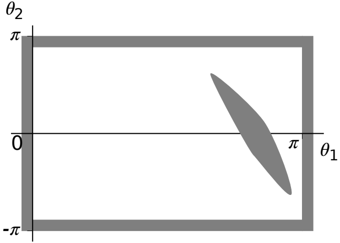
Workspace



First obstacle



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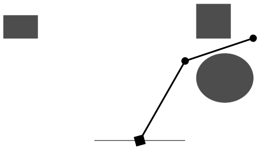
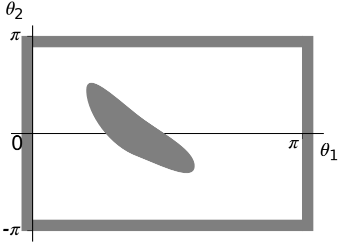
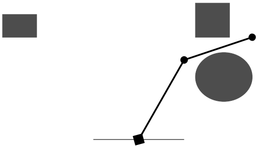


Second obstacle

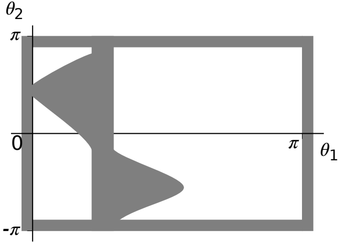


Robotic arm

Workspace

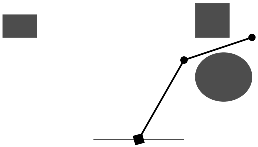


Third obstacle

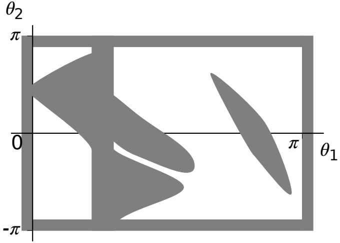


Robotic arm

Workspace



Free space



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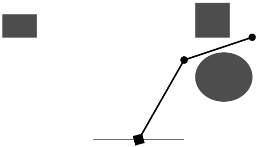

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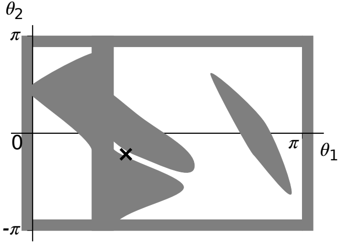


Conclusion on configuration space

Configuration space

- unification of mobile robots and robotic arms
- free space (no collision)
- adapted for planning

Building

- Minkowski sum for a mobile robot
- collision test for an arm

Limits

- only geometric constraints
- no kinematics nor dynamics
- potentially high-dimension space

02

Planning algorithms

Algorithms

Approaches

- space decomposition
  - grid
  - cell decomposition
- sampling
- potential fields
- geometric resolution
- path refinement

Path planning with a grid

Path planning with a grid

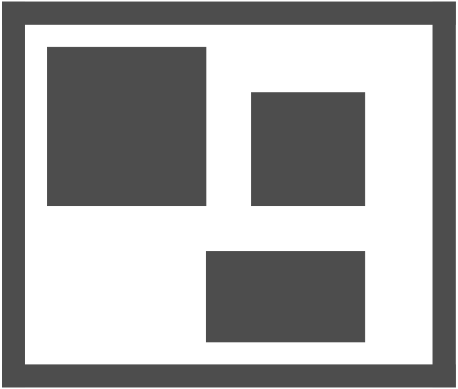
- adapted to occupancy grids
- neighborhood graph
- graph search (Dijkstra, A*)

Result

- path
- discretized orientation
- not necessarily optimal in distance
- high complexity in medium to high dimension

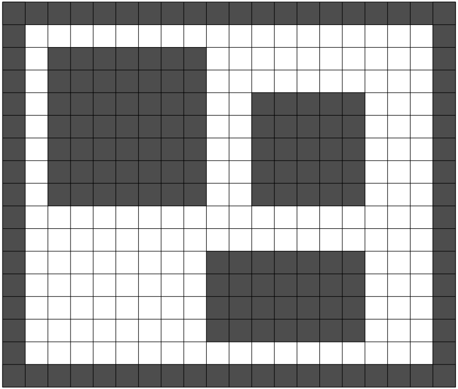
Example

Free space



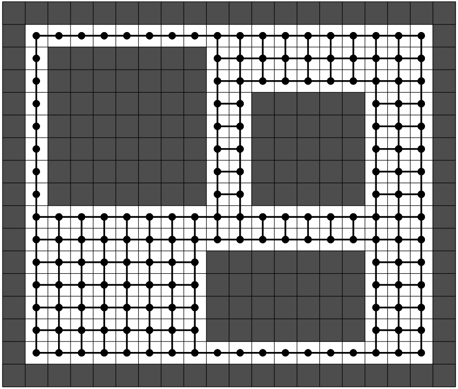
Example

Grid decomposition



Example

Cell graph



Example

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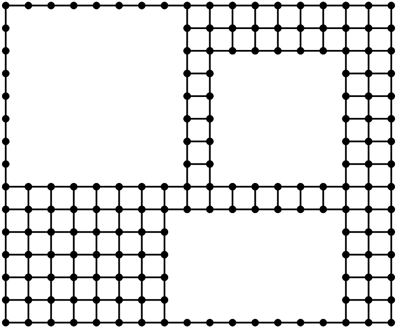
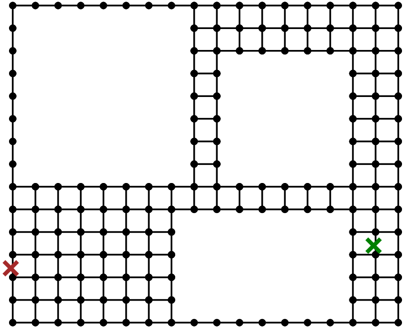


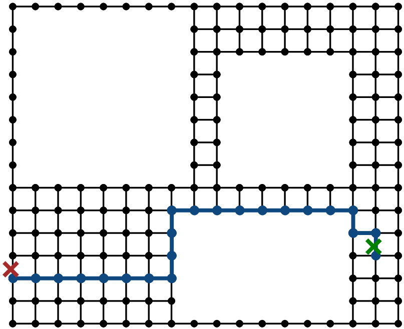
Example

Start and goal locations



Example

Graph search (A^* or other)



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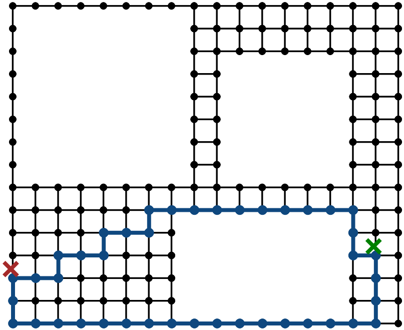
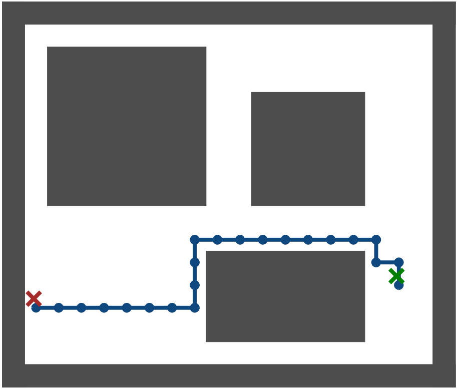


Shortest path not unique!

Example

Result



Planning with cell decomposition

Cell decomposition

- various tessellation methods

Voronoi diagram

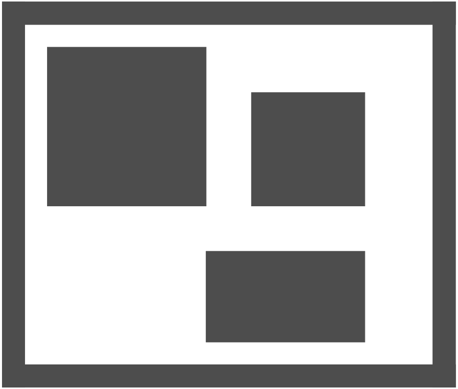
- tiling based on distance to obstacles
- dual of Delaunay triangulation
- path following of the edges of the cells

Result

- path
- as far from the obstacles as possible
- difficult to build in high dimensions
- not distance optimal

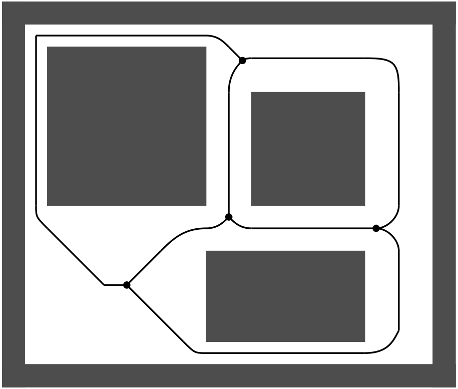
Example

Free space



Example

Voronoi diagram



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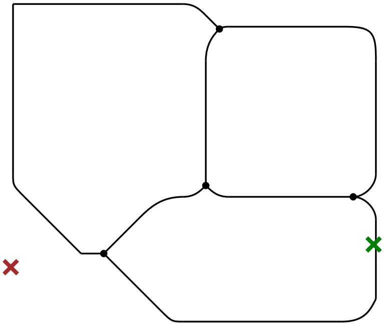
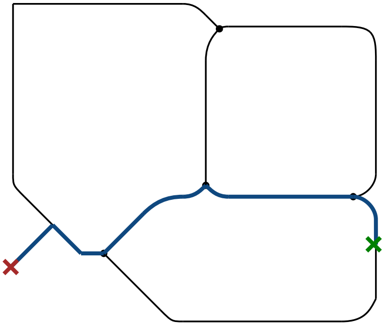


Example

Graph search



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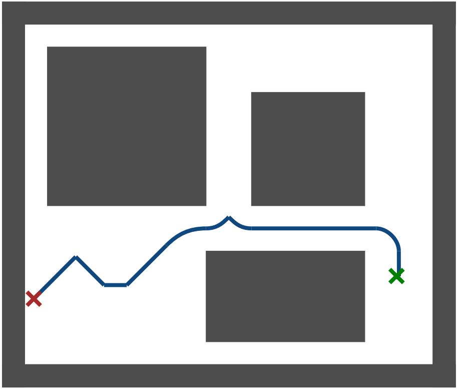


Sampling-based planning

Rapidly-expanding Random Trees

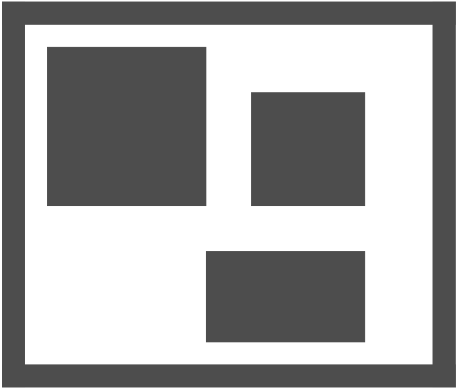
- RRT, RRT*...
- stochastic algorithm by sampling the space
- connection tree building
- until goal is found
- refinement of the path

Result

- path
- distance optimal for infinite samples
- quick and anytime as soon as the path is found

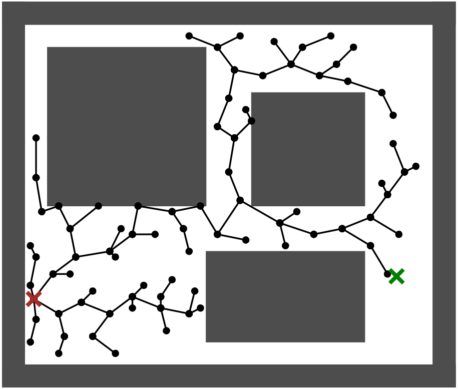
Example

Free space



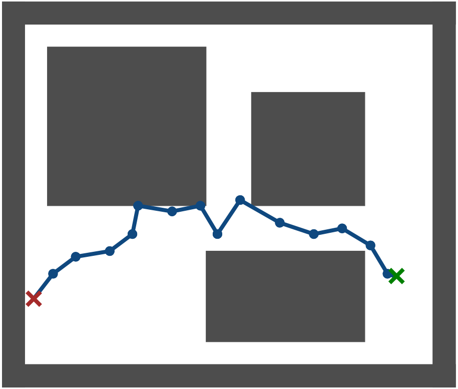
Example

Graph expansion



Example

Result



Algorithm

RRT

```
$V \leftarrow \{x_{\text{init}}\}; E \leftarrow \emptyset$
for $i = 1, \ldots, n$ do
    $x_{\text{rand}} \leftarrow \texttt{SampleFree}()$
    $x_{\text{nearest}} \leftarrow \texttt{Nearest}\left(G = (V, E), x_{\text{rand}}\right)$
    $x_{\text{new}} \leftarrow \texttt{Steer}\left(x_{\text{nearest}}, x_{\text{rand}}\right)$
    if $\texttt{CollFree}\left(x_{\text{nearest}}, x_{\text{new}}\right)$ then
        $V \leftarrow V \cup \{x_{\text{new}}\}$
        $E \leftarrow E \cup \{(x_{\text{nearest}}, x_{\text{new}})\}$
    end if
end for
return $G = (V, E)$
```

Functions

- $\texttt{SampleFree}()$: sample point in free space
- $\texttt{Nearest}(G, x)$: point in G nearest from x
- $\texttt{Steer}(x_1, x_2)$: point toward x_2 at a given distance from x_1
- $\texttt{CollFree}(x_1, x_2)$: no obstacle between x_1 and x_2

Algorithm

Improvement of RRT

```
$V \leftarrow \{x_{\text{init}}\}; E \leftarrow \emptyset$
for $i = 1, \dots, n$ do
    $x_{\text{rand}} \leftarrow \texttt{SampleFree}()$
    $x_{\text{nearest}} \leftarrow \texttt{Nearest}\left(G = (V, E), x_{\text{rand}}\right)$
    $x_{\text{new}} \leftarrow \texttt{Steer}\left(x_{\text{nearest}}, x_{\text{rand}}\right)$
    if $\texttt{CollFree}\left(x_{\text{nearest}}, x_{\text{new}}\right)$ then
        $X_{\text{near}} \leftarrow \texttt{Near}\left(G = (V, E), x_{\text{new}}, \delta\right)$
        $x_{\text{min}} \leftarrow \arg\min_{x \in X_{\text{near}}} \mathrm{C}(x) + c(x, x_{\text{new}})$
        $V \leftarrow V \cup \{x_{\text{new}}\}$
        $E \leftarrow E \cup \{(x_{\text{min}}, x_{\text{new}})\}$
    end if
end for
return $G = (V, E)$
```

Functions

- $\texttt{SampleFree}()$: sample point in free space
- $\texttt{Nearest}(G, x)$: point in G nearest from x
- $\texttt{Steer}(x_1, x_2)$: point toward x_2 at a given distance from x_1
- $\texttt{CollFree}(x_1, x_2)$: no obstacle between x_1 and x_2
- $\texttt{Near}(G, x, d)$: points in G at a distance from x less than d
- $\mathrm{C}(x)$: cost between x_{init} and x, walking up the graph
- $c(x_1, x_2)$: cost between x_1 and x_2

Algorithm

RRT*

```
$V \leftarrow \{x_{\text{init}}\}; E \leftarrow \emptyset$
for $i = 1, \ldots, n$ do
    $x_{\text{rand}} \leftarrow \texttt{SampleFree}()$
    $x_{\text{nearest}} \leftarrow \texttt{Nearest}\left(G = (V, E), x_{\text{rand}}\right)$
    $x_{\text{new}} \leftarrow \texttt{Steer}\left(x_{\text{nearest}}, x_{\text{rand}}\right)$
    if $\texttt{CollFree}\left(x_{\text{nearest}}, x_{\text{new}}\right)$ then
        $X_{\text{near}} \leftarrow \texttt{Near}\left(G = (V, E), x_{\text{new}}, \delta\right)$
        $x_{\text{min}} \leftarrow \arg\min_{x \in X_{\text{near}}} \mathrm{C}(x) + c\left(x, x_{\text{new}}\right)$
        $V \leftarrow V \cup \{x_{\text{new}}\}$
        $E \leftarrow E \cup \{(x_{\text{min}}, x_{\text{new}})\}$
        for all $x \in X_{\text{near}}$ do
            if $\mathrm{C}(x_{\text{new}}) + c\left(x_{\text{new}}, x\right) < \mathrm{C}(x)$ then
                $E \leftarrow E \setminus \{(\mathrm{P}(x), x)\}$
                $E \leftarrow E \cup \{(x_{\text{new}}, x)\}$
            end if
        end for
    end if
end for
return $G = (V, E)$
```

Functions

- $\texttt{SampleFree}()$: sample point in free space
- $\texttt{Nearest}(G, x)$: point in G nearest from x
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- $\mathrm{C}(x)$: cost between x_{init} and x, walking up the graph
- $c(x_1, x_2)$: cost between x_1 and x_2
- $\mathrm{P}(x)$: parent of x.

Planning with potential fields

Potential fields

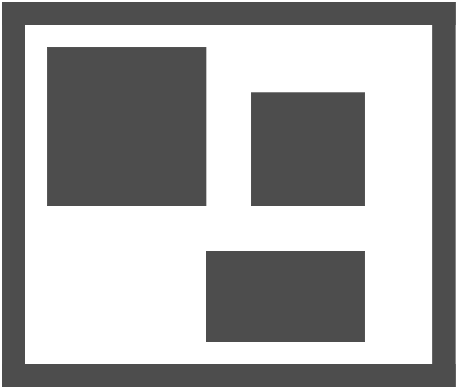
- repulsive field around obstacles
- attractive field around goal
- combination of both
- gradient descent

Result

- path
- away from obstacles
- quick to compute
- local minima

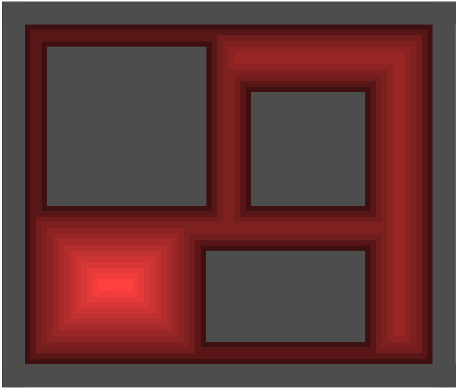
Example

Free space



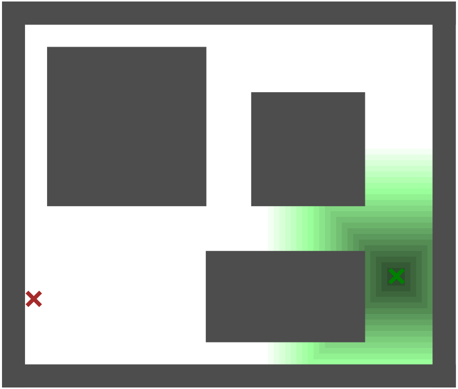
Example

Repulsive field



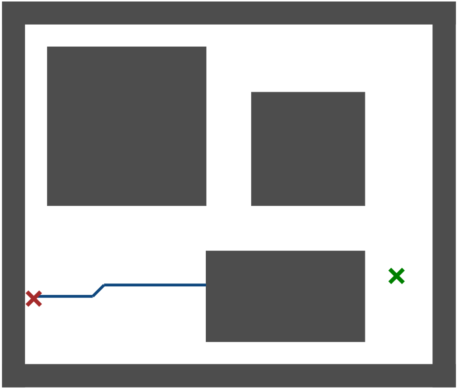
Example

Attractive field



Example

Gradient descent



Local minimum!

Planning with visibility graph

Visibility graph

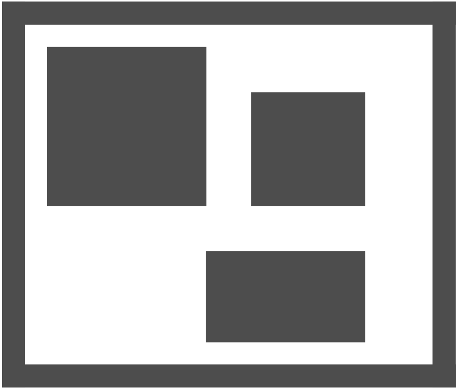
- nodes: vertices of obstacles
- edge: iff visibility between nodes
- start and goal as nodes
- graph search

Result

- path
- distance optimal
- can follow obstacle edges
- need polygonal/polyhedral obstacles

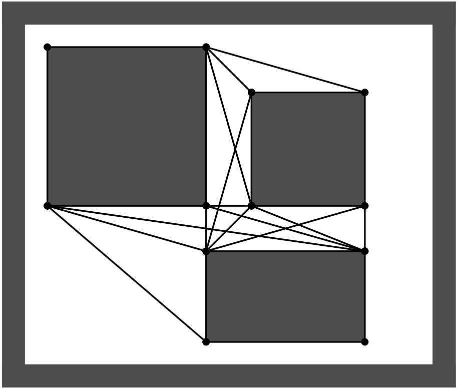
Example

Free space



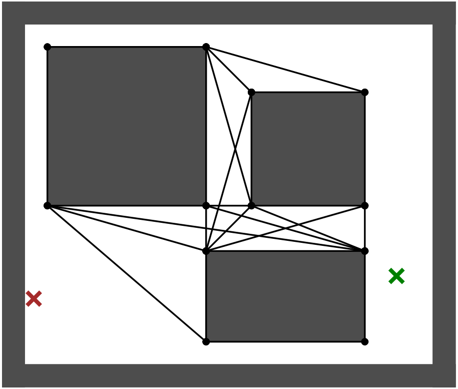
Example

Visibility graph of obstacles



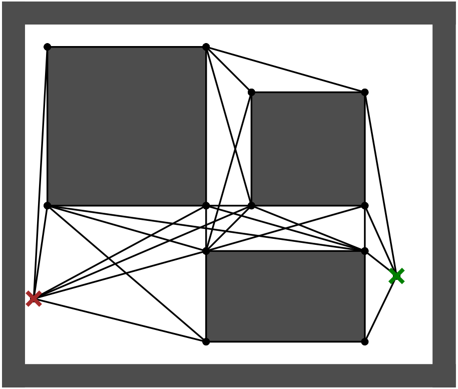
Example

Include start and goal



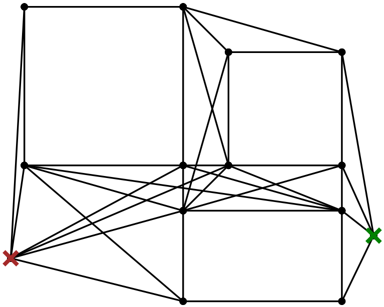
Example

Include start and goal



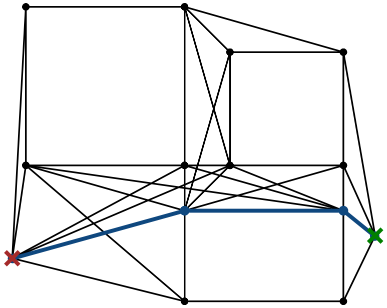
Example

Complete graph



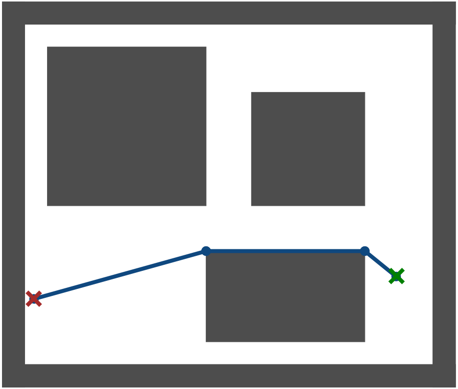
Example

Graph search (A* or other)



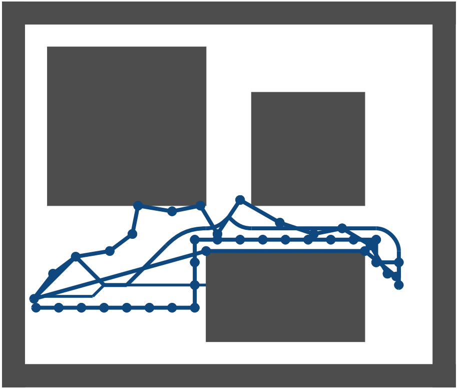
Example

Result



Comparison

Comparison of the paths



03

Conclusion

Conclusion

Configuration space

- space free from collision
- adapted to planning
- similar for mobile and articulated robots

Planning algorithms

- different families with different internal representations
- various optimization criteria

Limits

- known map
- static obstacles

Bibliography

RRT*, PRM*, etc.

- Karaman and Frazzoli, *Sampling-based algorithms for optimal motion planning*, IJRR, 2011.

Books

- Latombe, *Robot Motion Planning*, Kluwer Academic Publishers, 1991.
- Lavalle, *Planning Algorithms*, Cambridge University Press, 2006.
- Siciliano *et al.*, *Springer Handbook of Robotics*, 2nd ed., Springer, 2016.

Thanks for your attention

Questions?