Navigation

ST5 Autonomous robotics

Francis Colas

2022-10-07

Introduction

Path planning

- configuration space and planning algorithms
- known and static map

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- configuration space and planning algorithms
- known and static map

Navigation

- mobile robot motion
  - path planning
  - path execution
  - obstacle avoidance
- exploration
  - unknown environment
  - decide commands to build map

Introduction

Path planning

- configuration space and planning algorithms
- known and static map

Navigation

- mobile robot motion
- exploration

Aim of the session

- trajectory following
- obstacle avoidance
- exploration

01

Trajectory following

Trajectory following

Trajectory following

- decide commands to execute planned trajectory
- using sensor values
- taking into account the robot constraints

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General principle

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- given the position/error
- given the robot motion model
- compute a command to follow the trajectory

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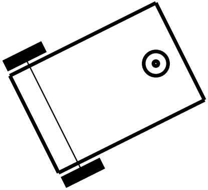
Kinematic models

Differential-drive robot

- left and right independent motor wheels
- caster wheel for stabilization



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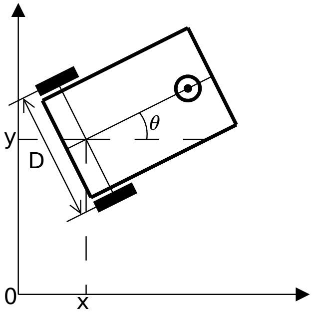

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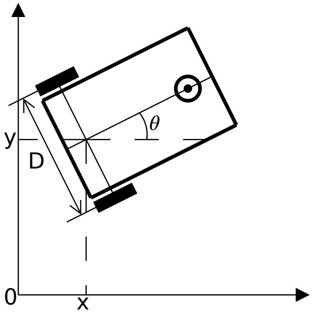
$$\begin{cases} \dot{x} &= \frac{v_r+v_l}{2} \cos\theta \\ \dot{y} &= \frac{v_r+v_l}{2} \sin\theta \\ \dot{\theta} &= \frac{v_r-v_l}{D} \end{cases}$$



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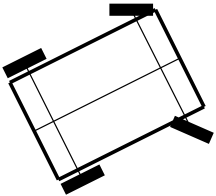
Kinematic models

Car-like vehicles

- front wheels can pivot
- rear wheels are fixed



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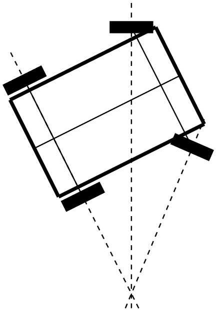

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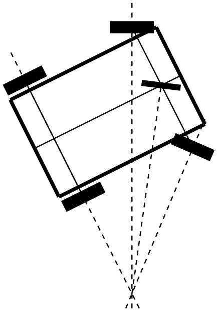

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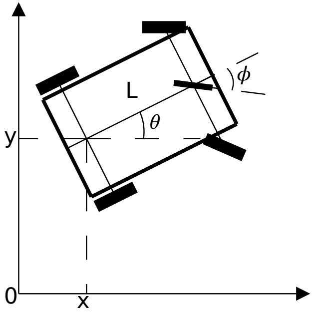

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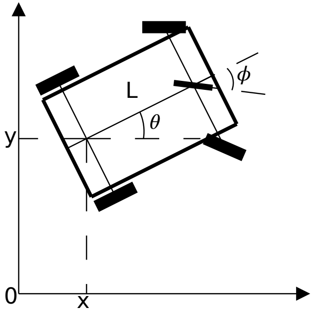
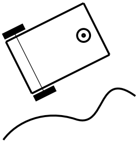


Trajectory following

Principle

- define commands as a function of error
- differential equation of error



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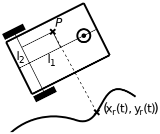

Differential-drive robot

- point motion

$$\begin{pmatrix} \dot{x}_P \\ \dot{y}_P \end{pmatrix} = \begin{pmatrix} \frac{v_r+v_l}{2} \cos\theta - \frac{v_r-v_l}{D}(l_1 \sin\theta + l_2 \cos\theta) \\ \frac{v_r+v_l}{2} \sin\theta - \frac{v_r-v_l}{D}(-l_1 \cos\theta + l_2 \sin\theta) \end{pmatrix} = \boldsymbol{M} \begin{pmatrix} v_r \\ v_l \end{pmatrix}$$

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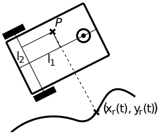
Trajectory following



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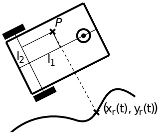

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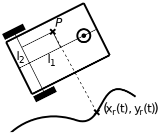

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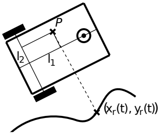

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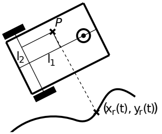

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$$\boldsymbol{u} = \boldsymbol{M}^{-1}\dot{\boldsymbol{x}}_r - \boldsymbol{M}^{-1}\boldsymbol{K}\boldsymbol{e}$$

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Conclusion on trajectory following

Trajectory following

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Proportional with feed-forward

- simple error reduction
- based on the kinematic model
- can be generalized to cars
- limits:
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Path following

- reference position
- projection in Frenet frame

02

Obstacle avoidance

Obstacle avoidance

Obstacle avoidance

- need exteroceptive sensors
- computation of new commands:
  - avoiding obstacles
  - while reaching target

Approaches

- potential fields
- vector field histogram
- dynamic window approach
- velocity obstacles

Vector field histogram

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- histogram of density of obstacles
- according to direction
- identification of density valleys
- choice of the deepest valley

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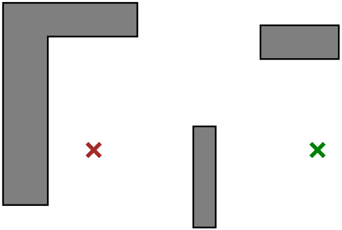

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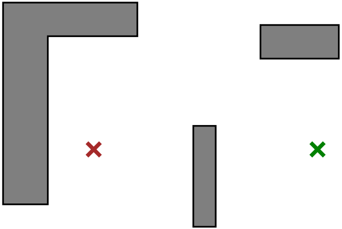

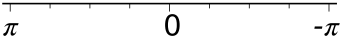

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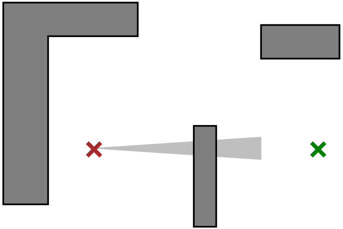

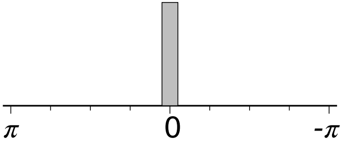

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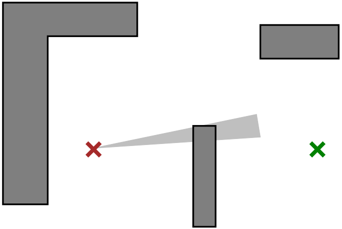

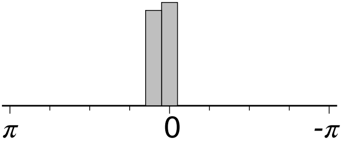

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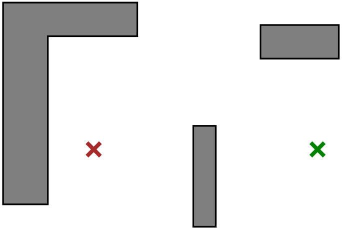

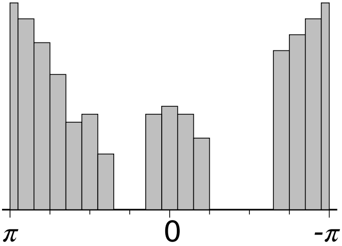

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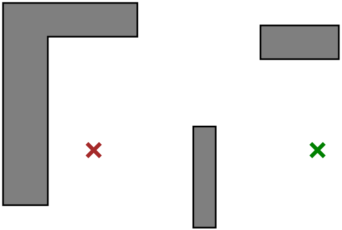

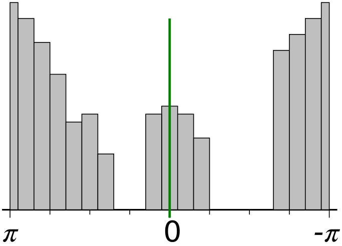

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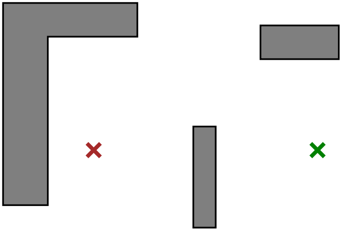

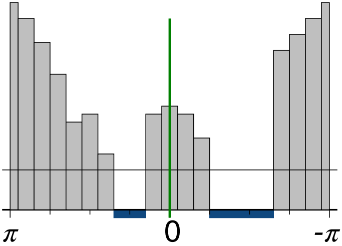

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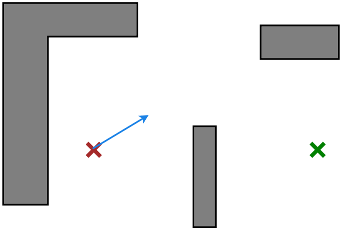

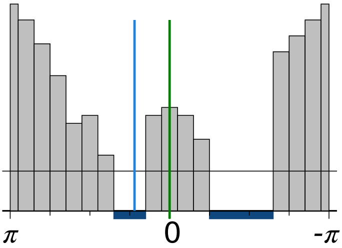

Dynamic window approach

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- command space
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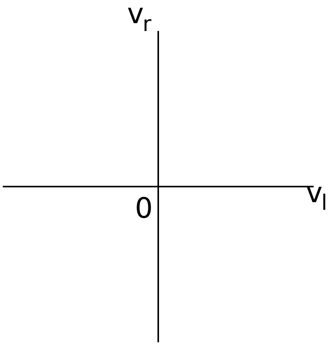

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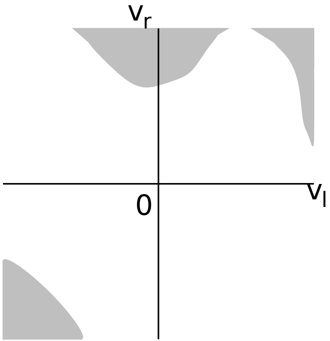

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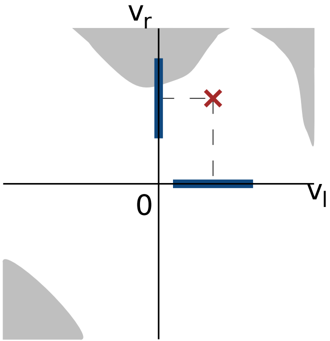

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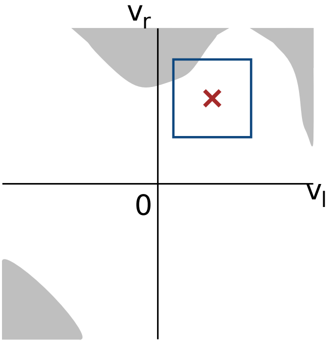

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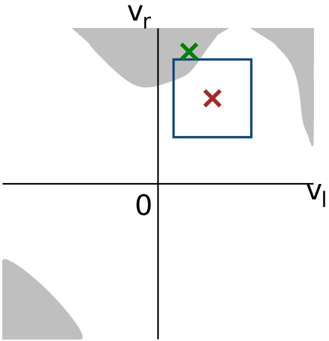

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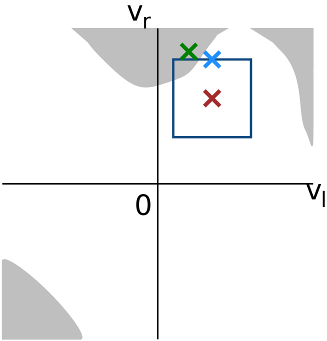

Dynamic window approach

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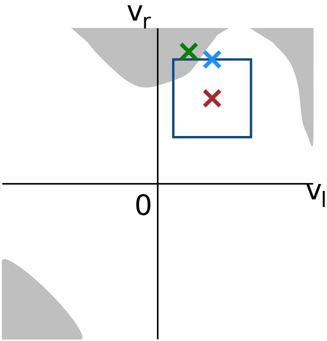

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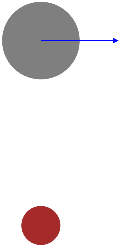

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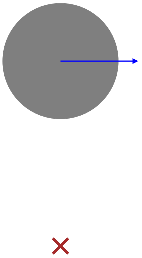

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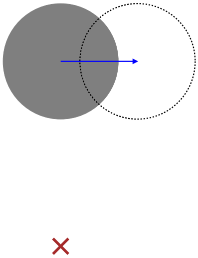

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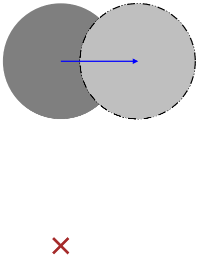

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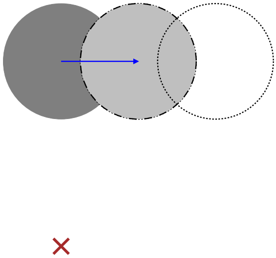

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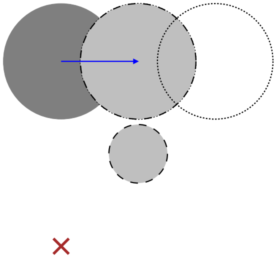

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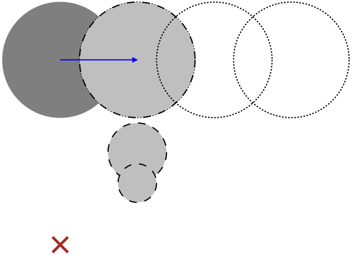

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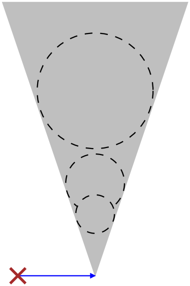

Conclusion on obstacle avoidance

Obstacle avoidance

- local modification of commands
- recognize acceptable commands
- fast reactions
- sometimes also trajectory following

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Limitations

- obstacle detection
- velocity estimation
- no general guarantee

03

Exploration

Autonomous motion decision

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- choose the actions of a mobile robot
- to discover an environment
- while building the map
- $\rightarrow$ information optimization

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- unknown localization
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Autonomous motion decision

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- choose the actions of a mobile robot
- to discover an environment
- while building the map
- $\rightarrow$ information optimization

Active localization

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Pursuit evasion problem

- find and follow another mobile object
- known or unknown environment

Information optimization

Information quantity

- use of entropy

$$H_p = \begin{cases} -\int p(x)\log p(x)\,\mathrm{d}x \\ -\sum_x p(x)\log p(x) \end{cases}$$

- entropy: uncertainty measurement
- maximize information by minimizing entropy

Information optimization

Information quantity

- use of entropy

$$H_p = \begin{cases} -\int p(x) \log p(x) \, \mathrm{d}x \\ -\sum_x p(x) \log p(x) \end{cases}$$

- entropy: uncertainty measurement
- maximize information by minimizing entropy

Information gain

- comparison between current and expected information

$$I_p(\boldsymbol{u}) = H_p - \mathsf{E}[H_{p'} \mid \boldsymbol{u}]$$

- unknown next observation

Exploration heuristics

Entropy

- correlation between entropy and information gain in an occupancy grid
- greedy method: choose best immediate action

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Uncertainty in unexplored space

- long-term plans are invalid
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Uncertainty in unexplored space

- long-term plans are invalid
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Frontier-based exploration

- list known borders
- go explore nearest

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Conclusion

Conclusion

Navigation

- motion decision
- adapt to robot: trajectory following
- adapt to environment: obstacle avoidance
- adapt to our knowledge: exploration

Conclusion

Navigation

- motion decision
- adapt to robot: trajectory following
- adapt to environment: obstacle avoidance
- adapt to our knowledge: exploration

Limits

- articulation between path planning and execution
- obstacle identification
- heuristic exploration

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Thanks for your attention

Questions?