Localization

ST5 Autonomous robotics

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Introduction

State estimation

- perception of the robot itself
- from sensor values
- with robot and sensor models

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Localization

- estimation of the pose (position, orientation, configuration)
- various kinds of maps
- various approaches

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- various approaches

Aim of the session

- definition of the pose
- localization
- maps

01

Pose

Kinds of robots

Mobile robots

- rigid-body in motion
- degrees-of-freedom: position and orientation of the mobile base

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Others

- parallel robots
- flexible robots...

Pose

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- parametrization allowing to derive the position of any point of the robot
- mobile base: position and orientation
- arm: angle/parameters of each joint between links

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- (at least) a frame per link (rigid object) + external/absolute frame
- bijection between relative pose and transformation

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Transformations

- rotations: special orthogonal group $\mathcal{SO}(n)$
- translations: vector space $\mathbb{R}^n$
- transformations: special Euclidean group $\mathcal{SE}(n)$

Pose space

Geometrical space

- position: $\mathbb{R}^3$ or $\mathbb{R}^2$
- orientation: $\mathcal{SO}(3)$ or $\mathcal{SO}(2)$
- pose
  - drone: $\mathbb{R}^3 \times \mathcal{SO}(3)$
  - ground robot: $\mathbb{R}^2 \times \mathcal{SO}(2)$
  - 3 dof arm: $\mathcal{SO}(2) \times \mathcal{SO}(2) \times \mathcal{SO}(2)$
- transformation
  - drone: $\mathcal{SE}(3)$
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Position parametrization

- vector $p \in \mathbb{R}^n$

Orientation parametrization

2D orientation

- $\mathcal{SO}(2)$ (*circle group*), 1-sphere S^1 (sometimess $\mathbb{S}^1$)
- unit complex: $\mathcal{T} = \{z \in \mathbb{C} : |z| = 1\}$
- rotation matrix (orthogonal, unitary, positive) : $\begin{pmatrix} \alpha & \beta \\ -\beta & \alpha \end{pmatrix}$
- "simplest" parametrization: angle $\theta \in \mathbb{R}/2\pi$

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3D orientation

- $\mathcal{SO}(3)$, 2-sphere S^2, or $\mathbb{S}^2$
- Euler angles: $\theta, \phi, \psi \in (\mathbb{R}/2\pi)^3 \rightarrow$ several conventions, *gimbal lock*
- rotation matrix
- axis-angle: $\mathbb{R}^3 \times \mathbb{R}/2\pi$
- unitary quaternions: $\{q \in \mathbb{H} : |q| = 1\}$

02

Localization

Localization

Definition

- find the pose of the robot
- with respect to a known map
- based on sensor values

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Approaches

- state estimation
- geometric constraint resolution
- error optimization

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State estimation approaches

- variants based on the representation
- Markov Localization
- Monte-Carlo localization

Markov Localization

Principle

- pose estimation
- based on odometry and exteroceptive observations
- Bayesian filter
- discretized state

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Inference

$$p(\boldsymbol{x}_k \mid \boldsymbol{z}_{1:k}, \boldsymbol{u}_{1:k})$$

$$p(\boldsymbol{x}_{k+1} \mid \boldsymbol{z}_{1:k}, \boldsymbol{u}_{1:k+1}) = \sum_{\boldsymbol{x}_k} p(\boldsymbol{x}_{k+1} \mid \boldsymbol{x}_k, \boldsymbol{u}_{k+1}) p(\boldsymbol{x}_k \mid \boldsymbol{z}_{1:k}, \boldsymbol{u}_{1:k})$$

$$p(\boldsymbol{x}_{k+1} \mid \boldsymbol{z}_{1:k+1}, \boldsymbol{u}_{1:k+1}) \propto p(\boldsymbol{z}_{k+1} \mid \boldsymbol{x}_{k+1}) p(\boldsymbol{x}_{k+1} \mid \boldsymbol{z}_{1:k}, \boldsymbol{u}_{1:k+1})$$

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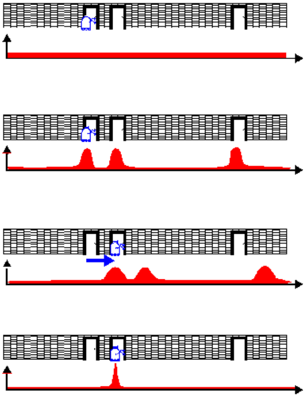
Markov Localization

Illustration



Markov Localization example (D. Fox)

Markov Localization

Example

- a robot in corridor with one door in $x_d = 6$
- $x_k \in \{0, \ldots, 9\}$, $u_k \in \{-1, 0, 1\}$, $z_k \in \{\text{far, close, in front}\}$

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- transition model
  - 90% chance arrive where expected
  - 5% chance to be too far or not far enough by 1 step

$$p(x_{k+1} \mid x_k, u_{k+1}, \boldsymbol{m}) = \begin{cases} 90\% & \text{if } x_{k+1} = x_k + u_{k+1} \\ 5\% & \text{if } \|x_{k+1} - (x_k + u_{k+1})\| = 1 \\ 0\% & \text{otherwise} \end{cases}$$

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- observation model
  - 80% chance to be correct
  - 10% chance for each other case

$$p(z_k \mid x_k, \boldsymbol{m}) = \begin{cases} 80\% & \text{if } \|x_k - x_d\| > 1 \text{ and } z_k = \text{far} \\ 80\% & \text{if } \|x_k - x_d\| = 1 \text{ and } z_k = \text{close} \\ 80\% & \text{if } x_k = x_d \text{ and } z_k = \text{in front} \\ 10\% & \text{otherwise} \end{cases}$$

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Example → where is the map?

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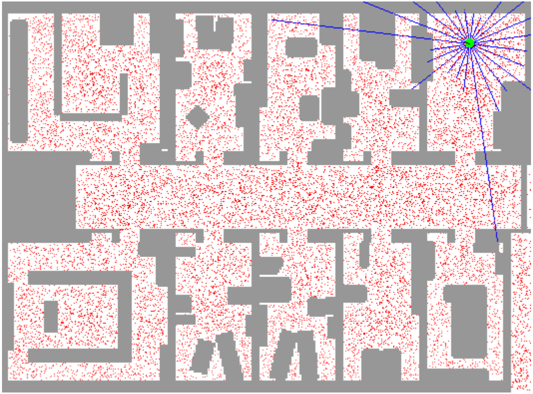
- resampling and loop

informatics mathematics

Monte Carlo Localization

Illustration [Fox, AAAI 1999]



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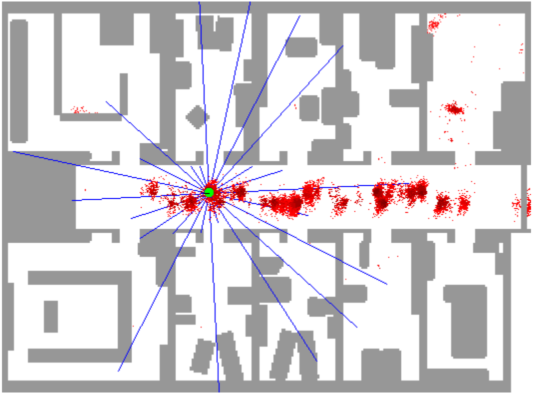

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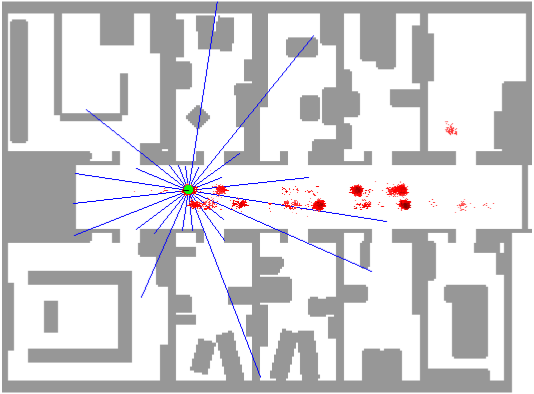

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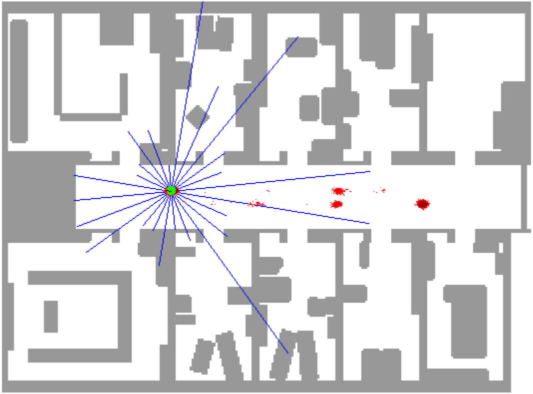

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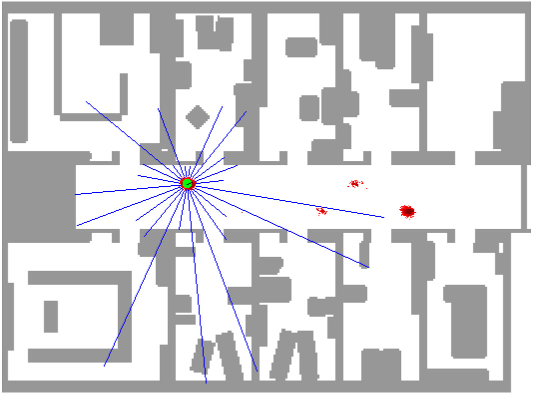

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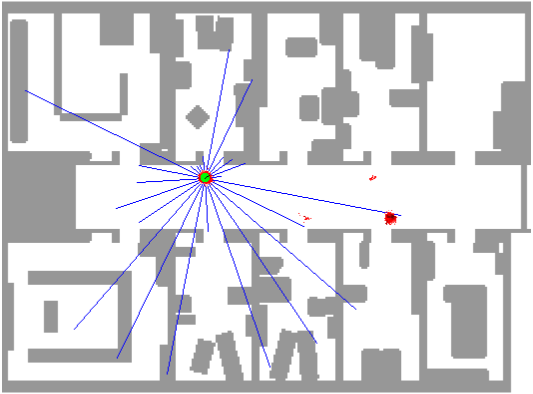

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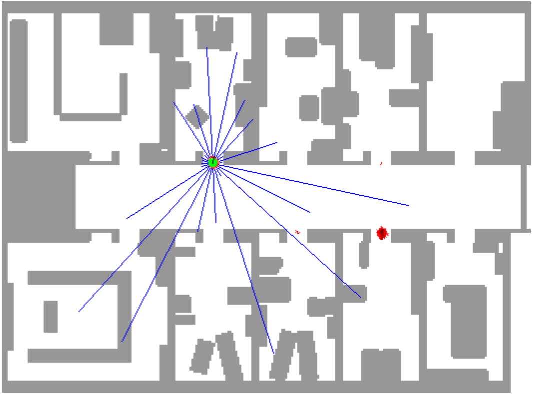

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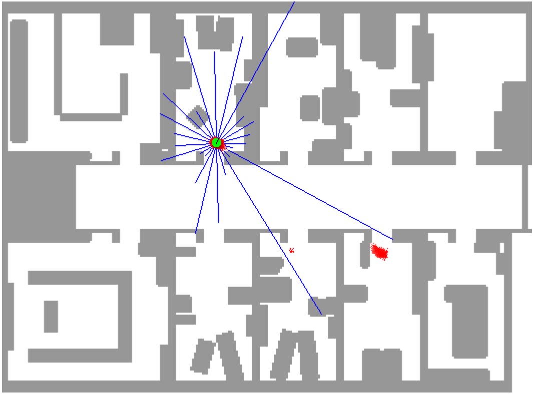

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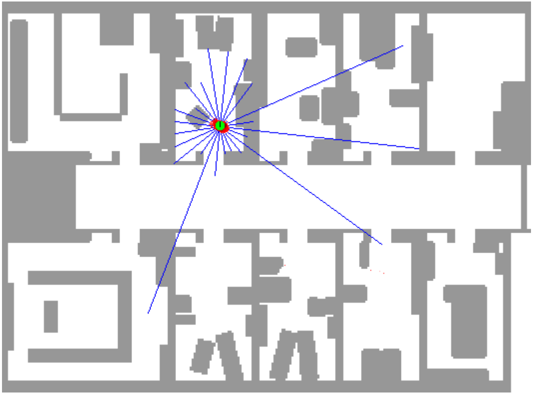

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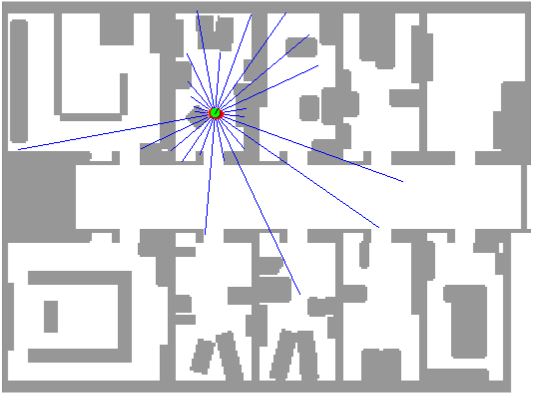

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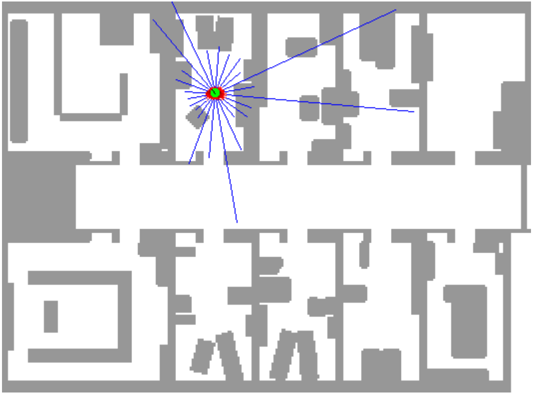

Conclusion on localization

Localization

- Markov localization
  - discretized distribution
  - bad scaling
- Monte Carlo localization
  - population of samples/particles
  - impoverishment and degeneracy issues
- known map

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03

Maps

Maps

Environment representation

- what is represented?
- characteristics?
- what to do with the maps?

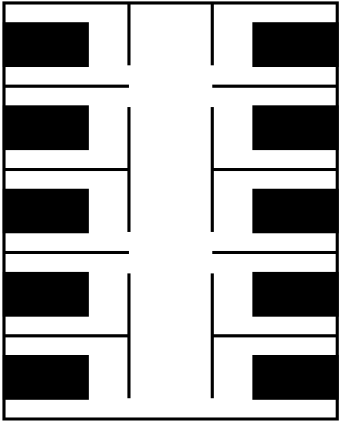
Maps

- geometric map
- landmark map
- point cloud map
- occupancy grid
- digital elevation map
- pose graph
- topological map
- semantic map

Geometric map

Geometric map

- set of geometric primitives
  - polygons or segments in 2D
  - triangles or polyhedra in 3D
- representation of obstacles
- metric dense map
- difficult to build from sensor values
- easy to plan a path
- scaling: number and complexity of obstacles

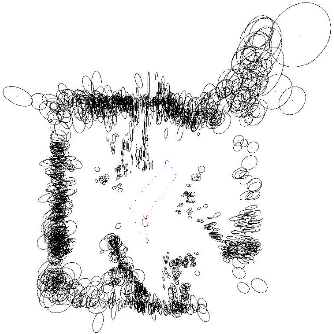


Geometric map

Landmark map

Landmark map

- set of landmarks
  - descriptor
  - position
- representation of salient elements
- metric sparse map
- easy to build from sensor values
- designed for localization
- very hard to plan a path
- scaling: number of landmarks

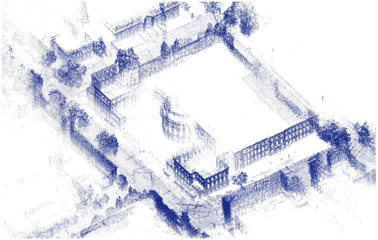


Visual landmark map

Point cloud map

Point cloud map

- set of points
  - position
  - sometimes color...
- representation of surface through sampling
- metric sparse map
- easy to build from sensor values
- hard to plan a path
- scaling: density of points and surfaces



Point cloud map

Occupancy grid

Occupancy grid

- dense representation of space
  - space segmentation
  - probability of occupancy
- representation of free, occupied of unknown space
- metric dense map
- easy to build from distance sensor values
- easy to plan a path
- scaling: surface (or volume) and resolution

Occupancy grid

Digital elevation map

Digital elevation map

- representation of height of the ground
- 2.5D map
- metric dense map
- ok to build from distance sensor values
- easy to plan a path
- scaling: surface and resolution

Digital elevation map

Pose graph

Pose graph

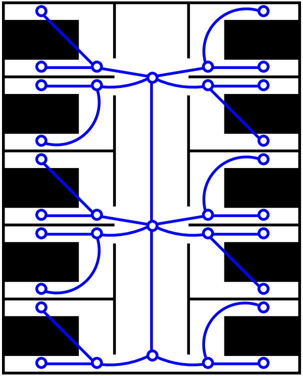
- list of poses of robots
  - sensor values
  - link between poses
- structuring of sensor values
- non metric map
- easy to build
- hard to plan a path
- scaling: size of sensor data

Pose graph

Topological map

Topological map

- graph of zones/places
  - empty space
  - neighbor relationship between zones
- higher level
- non metric map
- hard to plan a path
- scaling: number of places



Geometric and topological map

Topological map

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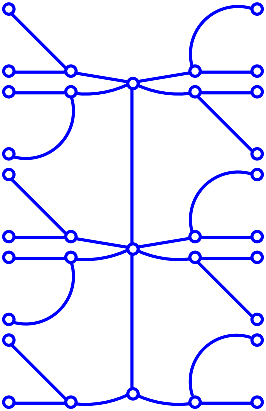
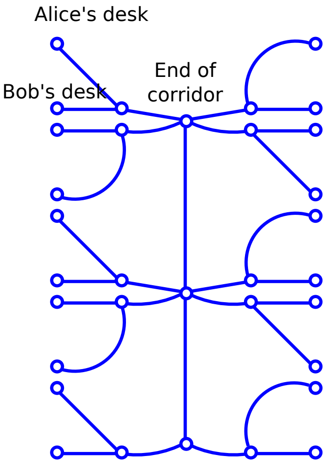


Topological map

Semantic map

Semantic map

- semantic information
  - qualitative
  - usage
- non metric map
- very hard to build from sensor data
- high-level planning
- scaling: number of places



Semantic map

Conclusion on maps

Various uses

- localization
- path planning
- task planning
- visualization

Various characteristics

- metric or not
- dense or not
- ease of building from sensor values
- ease of path planning

04

Conclusion

Conclusion

Robot pose

- parametrization of position of all points of the robot
- position, orientation, joints
- orientation in 3D is tricky

Localization

- estimate the pose of the robot with a known map
- various approaches
- various algorithms

Maps

- various kinds of map
- choice depends on usage

Bibliography

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Thanks for your attention

Questions?